2 0 1 6

(4th Semester)

MATHEMATICS

Paper : MATH–241

(Vector Calculus and Solid Geometry)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Prove by vector method

sin( ) sin cos cos sin

for any two angles and . 3

(b) Find a vector whose magnitude is 3 units and which is perpendicular to each of the vectors $\vec{a}\ \ 3\hat{i}\ \ \hat{j}\ \ 4\hat{k}$ and $\vec{b}\ \ 6\hat{i}\ \ 5\hat{j}\ \ 2\hat{k}$. 4

(c) Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{d}_1\ \ 3\hat{i}\ \ \hat{j}\ \ 2\hat{k}$ and $\vec{d}_2\ \ \hat{i}\ \ 3\hat{j}\ \ 4\hat{k}$. 3

2. *(a)* Prove that

$$[(\vec{a}\ \ \vec{b})(\vec{b}\ \ \vec{c})(\vec{c}\ \ \vec{a})]\ \ 2[\vec{a}\,\vec{b}\,\vec{c}]$$ 3

(b) Prove that a vector function $\vec{f}(t)$ to be a constant magnitude if and only if

$$\vec{f}\ \frac{d\vec{f}}{dt}\ \ 0$$ 4

(c) A particle moves along the curve $x\ \ 2t^2$, $y\ \ t^2\ \ 4t$, $z\ \ 3t\ \ 5$, where t is the time. Find the velocity and acceleration at time $t\ \ 1$. 3

UNIT—II

3. *(a)* Find the unit outward drawn normal to the surface $(x\ \ 1)^2\ \ y^2\ \ (z\ \ 2)^2\ \ 9$ at the point $(3,\ 1,\ \ 4)$. 3

(b) Let $(x, y, z)\ \ x^3\ \ y^3\ \ z^3$. Find the directional derivative of at $(1,\ \ 1,\ 2)$ in the direction of the vectors $\hat{i}\ \ 2\hat{j}\ \ \hat{k}$. 3

(c) Suppose $\vec{A} \quad \vec{0}$. Evaluate $(\vec{A} \quad \vec{r}) \quad 0$, where $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$ and $\vec{A} \quad A_1\hat{i} \quad A_2\hat{j} \quad A_3\hat{k}$. 4

4. (a) Show that

$$_S \vec{F} \quad \vec{n}\, dS \qquad _v \quad \vec{F}\, dv$$

where $\vec{F} \quad 4xz\hat{i} \quad y^2\hat{j} \quad yz\hat{k}$ and S is the surface of the cube bounded by $x \quad 0$, $x \quad 1$, $y \quad 0$, $y \quad 1$, $z \quad 0$, $z \quad 1$. 5

(b) Find the work done in moving a particle in the force field

$$\vec{F} \quad 3x^2\hat{i} \quad (2xz \quad y)\hat{j} \quad z\hat{k}$$

along *(i)* a straight line from (0, 0, 0) to (2, 1, 3) and *(ii)* the curve defined by $x^2 \quad 4y$, $3x^2 \quad 8z$ from $x \quad 0$ to $x \quad 2$. 5

Unit—III

5. (a) If by rotation of the axes about the origin, the expression $ax^2 \quad 2hxy \quad by^2$ changes to $a x^2 \quad 2h x y \quad b y^2$, then show that $a \quad b \quad a \quad b$ and $ab \quad h^2 \quad a b \quad h^2$. 4

(b) Prove that the equation

$$2x^2 \quad 5xy \quad 3y^2 \quad 2x \quad 3y \quad 0$$

represents a pair of straight lines. Find the coordinates of their point of intersection and the angle between them. 2+2+2=6

6. (a) Reduce the equation

$$14x^2 \quad 4xy \quad 11y^2 \quad 44x \quad 58y \quad 71 \quad 0$$

to the standard form. 5

(b) Find the equation of the chord of contact of tangents from a given point (x, y) to the conic

$$ax^2 \quad 2hxy \quad by^2 \quad 2gx \quad 2fy \quad c \quad 0$$ 5

Unit—IV

7. (a) Find the equation of the plane which passes through the point (2, 1, 4) and is perpendicular to the planes

$$9x \quad 7y \quad 6z \quad 48 \quad 0$$

and $x \quad y \quad z \quad 0$. 5

(b) If a plane cuts the axes at A, B, C and the centroid of the ABC is (a, b, c), then prove that the equation of the plane is

$$\frac{x}{a} \quad \frac{y}{b} \quad \frac{z}{c} \quad 3$$ 5

8. *(a)* Prove that the lines

$$\frac{x \quad 1}{2} \quad \frac{y \quad 2}{3} \quad \frac{z \quad 3}{4}$$

and $\quad 4x \quad 3y \quad 1 \quad 0 \quad 5x \quad 3z \quad 2 \quad$ are coplanar. 5

(b) Find the length and equation of the line of the shortest distance between the lines

$$\frac{x \quad 3}{4} \quad \frac{y \quad 6}{3} \quad \frac{z}{2} \text{ and } \frac{x \quad 2}{4} \quad \frac{y}{1} \quad \frac{z \quad 7}{1}$$ 5

UNIT—V

9. *(a)* Find the centre and radius of the sphere given by

$$x^2 \quad y^2 \quad z^2 \quad 4x \quad 5y \quad 6z \quad 1 \quad 0$$ 5

(b) Find the equation of the sphere which passes through the origin and touches the sphere $x^2 \quad y^2 \quad z^2 \quad 56$ at the point $(2, \quad 4, 6)$. 5

10. *(a)* Find the equation of the line of intersection of the plane $3x \quad 4y \quad z \quad 0$ and the cone $15x^2 \quad 32y^2 \quad 7z^2 \quad 0$. 5

(b) Find the equation of a cylinder whose generating lines have the direction cosines l, m, n and which passes through the circle $x^2 \quad z^2 \quad a^2$, $y \quad 0$. 5

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Subject Code : MATH/IV/04

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 4th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

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DEGREE 4th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

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MATH/IV/04

2 0 1 6

(4th Semester)

MATHEMATICS

Paper : MATH–241

(Vector Calculus and Solid Geometry)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. The value of $[\hat{i}\ \hat{k}\ \hat{j}]$ is

(a) 1 ☐

(b) 1 ☐

(c) 0 ☐

(d) None of the above ☐

2. The projection of $\vec{a} \quad 2\hat{i} \quad \hat{j} \quad \hat{k}$ and $\vec{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$ is

(a) $\frac{5}{6}(2\hat{i} \quad \hat{j} \quad \hat{k})$ ☐

(b) $\frac{6}{5}(2\hat{i} \quad \hat{j} \quad \hat{k})$ ☐

(c) $\frac{6}{5}(\hat{i} \quad 2\hat{j} \quad \hat{k})$ ☐

(d) $\frac{5}{6}(\hat{i} \quad 2\hat{j} \quad \hat{k})$ ☐

3. If $\vec{a}$ is a constant function, then $\vec{a}$ is

(a) both solenoidal and irrotational ☐

(b) solenoidal ☐

(c) irrotational ☐

(d) neither solenoidal nor irrotational ☐

4. If $\vec{a}$ is any vector and $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$, then $(\vec{a} \quad)\vec{r}$ is

(a) $\vec{a}$ ☐

(b) $\vec{r}$ ☐

(c) $\vec{a} \quad \vec{r}$ ☐

(d) None of the above ☐

5. The equation of the circle $(x \quad h)^2 \quad (y \quad k)^2 \quad r^2$, when the origin is transferred to the point (h, k) is

(a) $x^2 \quad y^2 \quad r^2$ ☐

(b) $x^2 \quad y^2 \quad r^2$ ☐

(c) $(x \quad h)^2 \quad y^2 \quad r^2$ ☐

(d) $x^2 \quad (y \quad k)^2 \quad r^2$ ☐

6. If m_1 and m_2 are the slopes of the two lines that the equation $ax^2 \quad 2hxy \quad by^2 \quad 0$ represents, then m_1 and m_2 are connected by the relation

(a) $m_1 \quad m_2 \quad \frac{2h}{a}$ ☐

(b) $m_1 m_2 \quad \frac{b}{a}$ ☐

(c) $m_1 \quad m_2 \quad \quad \frac{2h}{a}$ ☐

(d) $m_1 m_2 \quad \frac{h}{a}$ ☐

7. The equation of the plane through the point (2, 3, 5) and parallel to the plane $2x \quad 4y \quad 3z \quad 9$ is

(a) $2x \quad 4y \quad 3z \quad 7$ ☐

(b) $2x \quad 4y \quad 3z \quad 7$ ☐

(c) $2x \quad 4y \quad 3z \quad 9$ ☐

(d) $2x \quad 4y \quad 3z \quad 9$ ☐

8. The coordinates of the point of intersection of the line

$$\frac{x \quad 1}{1} \quad \frac{y \quad 3}{3} \quad \frac{z \quad 2}{2}$$

with the plane $3x \quad 4y \quad 5z \quad 5$ are

(a) $(r \quad 1, 3r \quad 3, 2r \quad 2)$ ☐

(b) $(r \quad 1, 3r \quad 3, 2r \quad 2)$ ☐

(c) $(r \quad 1, \quad 3r \quad 3, 2r \quad 2)$ ☐

(d) $(r \quad 1, 3r \quad 3, \quad 2r \quad 2)$ ☐

9. The equation of a sphere which passes through the origin and makes equal intercepts of unit length of the axes is

(a) $(x \quad 1)^2 \quad (y \quad 1)^2 \quad (z \quad 1)^2 \quad 0$ ☐

(b) $x^2 \quad y^2 \quad z^2 \quad r^2$ ☐

(c) $(x \quad 1)^2 \quad (y \quad 1)^2 \quad (z \quad 1)^2 \quad 0$ ☐

(d) $x^2 \quad y^2 \quad z^2 \quad 1$ ☐

10. The general equation of a curve which touches the three coordinate planes is

(a) $fx \mp gy \mp hz \quad 0$ ☐

(b) $\sqrt{fx} \mp \sqrt{gy} \mp \sqrt{hz} \quad 0$ ☐

(c) $fgh \mp xyz \quad 0$ ☐

(d) $fy \mp gx \mp hz \quad 0$ ☐

where f, g, h are parameters and carry their usual meaning.

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

State *True* or *False* by putting a Tick ☑ mark in the box provided and give a brief justification :

1. The vector of magnitude 6 which is perpendicular to both the vectors $\vec{a}\quad 4\hat{i}\quad \hat{j}\quad 3\hat{k}$ and $\vec{b}\quad 2\hat{i}\quad \hat{j}\quad 2\hat{k}$ is $(\ \hat{i}\quad 2\hat{j}\quad 2\hat{k})$.

True ☐ *False* ☐

Justification :

2. ($\vec{F}$) 0.

True ☐ *False* ☐

Justification :

3. The equation of the diameter of the conic $4x^2 \quad 6xy \quad 5y^2 \quad 1$ conjugates to the diameter $y \quad 2x$ is $10x \quad 7y \quad 0$.

True ☐ *False* ☐

Justification :

4. The condition that the line

$$\frac{x \quad x_1}{l} \quad \frac{y \quad y_1}{m} \quad \frac{z \quad z_1}{n}$$

is parallel to the plane $ax \quad by \quad cz \quad d \quad 0$ is $al \quad bm \quad cn \quad 0$.

True ☐ *False* ☐

Justification :

5. The equation

$$4x^2 \quad y^2 \quad 2z^2 \quad 2xy \quad 3yz \quad 12x \quad 11y \quad 6z \quad 4 \quad 0$$

represents a cone with vertex (1, 2, 3).

True ☐ *False* ☐

Justification :

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